pressure increases. After entering the heating section the fluid becomes less dense and the behavior in this region is similar to that given for small D.

For the transient study, calculations were begun at time $\tau = 0$, for a specified velocity, w_i, in the counter-clockwise direction and a temperature, ϕ_i. The finite-difference method of ref. [3] was used. The temperature and the velocity were solved simultaneously from the integrated form of equation (1) around the loop and from equation (2). The resulting values were then substituted into equation (3) to obtain the pressure. Typical results are given in Fig. 3(b).

For small time, the hot fluid leaving the heating section has only moved a small distance in the cooling section. Therefore, it is only within a small 'penetration depth' near $\theta = 0^+$ that heating assists the flow *in the cooling section.* Beyond this region $\phi = \text{const.} = \phi_i = 0$ in the cooling section and the pressure decreases until $\theta \approx \pi/2$. Beyond $\theta \approx \pi/2$ the flow is aided by gravity. In the heating section the cold fluid has only penetrated to a region near $\theta = \pi^+$ and in this region the flow is aided. Outside this region, $\phi = \text{const.}$ and the flow is opposed up to $\theta \approx 3\pi/2$ and aided over $3\pi/2 < \theta < 2\pi$.

While the variation of the dynamic pressure is of interest, the ratio of the maximum dynamic pressure to the maximum hydrostatic pressure is of greater significance, namely

$$\left|\frac{\Delta p_{d_{max}}}{\Delta p_{s_{max}}}\right| \simeq \frac{\Gamma \rho_w V^2/2}{\rho_w g R} = 10.0\left(\frac{\beta R q \mu}{\rho_w^2 g r^3 c}\right)^{1/2} = 4.5\times 10^{-6} q^{1/2}. \quad (15)$$

For the experimental system of refs. [1, 2], $R = 0.38$ m, $r = 0.015$ m, and q varies from 1000 to 4000 W m^{-2} for stable runs [6]. Water properties are evaluated at $T = 27$°C. Thus, the static pressure is up to four orders of magnitude greater than the dynamic pressure. Therefore, the total pressure is essentially hydrostatic.

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HIGH RAYLEIGH NUMBER HEAT TRANSFER IN A HORIZONTAL CYLINDER WITH ADIABATIC WALL

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NOMENCLATURE

A	cross-sectional area of cylinder [m^2]
C	Gill's free constant, dimensionless
d'	dimensionless width of thermal boundary layer, $1/m$
d	width of thermal boundary layer, $d'L$ [m]
g	gravitational acceleration [m s^{-2}]
h	height of rectangular enclosure [m]
k	fluid thermal conductivity [W m^{-1} K^{-1}]
k_1	dimensionless axial temperature gradient in cylinder core region
L	length of cylinder, or width of rectangular 2-D enclosure [m]
m	constant, dimensionless, equation (2)

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Nu	Nusselt number, equation (4)
Pr	Prandtl number, ν/κ
Q	net heat transfer rate [W]
r_0	radius of cylinder [m]
Ra	Rayleigh number for cylinder, $\beta g \Delta T r_0^3/\nu\kappa$
Ra_e	Rayleigh number for rectangular enclosure, $\beta g \Delta T (h/2)^3/\nu\kappa$
s	width of momentum boundary layer, Fig. 1 [m]
T	temperature [K]
T'	dimensionless temperature, equation (3)
T_h	temperature of hot thermode [K]
T_c	temperature of cold thermode [K]
ΔT	$T_h - T_c$ [K]
x, y, z	coordinates, Fig. 1 [m].

Greek symbols

β	coefficient of volumetric thermal expansion [K^{-1}]
δ	extent of end region [m]
κ	fluid thermal diffusivity [$m^2 s^{-1}$]
ν	kinematic viscosity [$m^2 s^{-1}$].

INTRODUCTION

FREE convection heat transfer in shallow rectangular enclosures has received considerable attention in recent years [1–14]. As a result, Nusselt numbers, Nu, in such geometries can be theoretically predicted rather well.

Relatively little, however, is known about the heat transfer in long horizontal cylinders with differentially heated end walls (thermodes) [15–18]. Bejan and Tien [15] obtained an analytical expression for velocity and temperature distributions in the core region of a cylinder by carrying out a perturbation analysis in the Rayleigh number, Ra, to three terms. They presented an expression for Nu as a function of Ra, obtained by integrating the convective and conductive contributions to the overall heat transfer over a cross-sectional area in the core. Kimura and Bejan [16] numerically determined velocity and temperature distributions but did not calculate Nu. The same authors performed an experimental study of free convection in a horizontal cylinder with different end temperatures [17]. They observed the circulation of water in a Plexiglas cylinder (length $L = 124.5$ cm, radius $r_0 = 7$ cm). The Ra-range was approximately 10^7–10^9, where $Ra \propto r_0^3$. Probing the temperature field with thermistors, these authors found a linearly stratified core which covered almost the entire vertical diameter. Furthermore, they measured the net axial heat transfer rate as a function of Ra, and presented the experimental results together with Bejan and Tien's [15] theoretical prediction. The theory overestimated Nu by several orders of magnitude.

Shih [18] extended Bejan and Tien's [15] three-term expansion for velocity and temperature distributions in the core region to 47th order in Ra. He obtained a 24-term series in Ra^2 for Nu and, after analyzing its singularities, presented an analytical expression for Nu supposedly valid for any Ra. This expression contains the dimensionless axial temperature gradient k_1 in the core of the cylinder. Shih estimated k_1 by matching Bejan and Tien's [15] first-order core solution with an integral solution for velocity and temperature in the end regions. He also calculated Nu as a function of Ra for several aspect ratios. His predictions lie almost two orders of magnitude below Kimura and Bejan's [17] experimental data.

In this note an experimentally guided approach is taken in estimating Nu at high Ra for a cylinder with an adiabatic side wall. We have recently measured the velocity distribution of free convective gas flows in a horizontal cylinder ($r_0/L = 0.10$, $r_0 = 1.0$ cm) [19]. There we have shown that Gill's theory [20], originally designed to describe the flow at the differentially heated vertical walls in tall rectangular enclosures, also predicts rather well the location and magnitude of the velocity maximum of the flow parallel to the vertical end walls in a long cylinder. Similar agreement was found for shallow rectangular enclosures by Simpkins and Dudderar [13]. Based on this good agreement for momentum transfer, we assume that at high Ra also the temperature gradient at the end walls of a cylinder can be estimated from Gill's treatment. Furthermore, we utilize the fact that at high Ra, fluids with not too small a Prandtl number, Pr, become thermally stratified in the enclosure geometries under consideration. Hence, we can estimate Nu by calculating the heat transport from a thermally stratified fluid through the thermal boundary layer to the end wall. The results of our calculations are presented here and agree well with Kimura and Bejan's [17] experimental data.

EXPERIMENTAL AND THEORETICAL BACKGROUND

So far no attempt has been made to predict Nu's in cylinders via a boundary layer approach. This is probably due to two reasons. First, the governing equations for the cylindrical geometry are much more complicated than the simplified 2-D equations used to derive Nu in rectangular enclosures (see e.g. the boundary layer approach in ref. [2]). Second, no experimental work had been done to characterize the flow reversal in the end zones of a cylinder. Our concurrent work [19], however, reveals some characteristic details which are used here to simplify Nu estimates. The relevant experimental findings in ref. [19] can be summarized as follows: the maxima of the vertical velocity component lie, for $Ra \gtrsim 5 \times 10^4$, in planes that are inclined with respect to the vertical end walls. For the region at the cold end of the cylinder, where the flow in the upper half is directed towards the end wall, this condition is schematically shown in Fig. 1 and presented in greater detail in ref. [19]. The distance s at mid-height (i.e. at $z = r_0$, see Fig. 1) and the magnitude of the vertical velocity maximum at $x = s$, $y = 0$, $z = r_0$, were found to be well predicted by Gill's theory for tall rectangular 2-D enclosures [20].

Gill's results were presented in a particularly revealing form by Roux *et al.* [21]. The dimensionless temperature distribution at mid-height of the vertical cold end wall in a rectangular cavity can be written, according to these authors, as

$$T' = \tfrac{1}{2}[-\exp(-mx/L)\cos(mx/L) + 1], \qquad (1)$$

with

$$m = \frac{0.841}{C}\frac{L}{h} Ra_e^{0.25}, \qquad (2)$$

and

$$T' = \frac{T - T_c}{T_h - T_c}, \qquad (3)$$

with the other parameters as listed in the nomenclature. Corresponding to our use of r_0 as the length scale in Ra, we used $h/2$ in Ra_e. Gill's free constant C was numerically determined by Roux *et al.* as 0.80 for the case of the square cavity [21] and was experimentally found to be comparable for long cylinders [19].

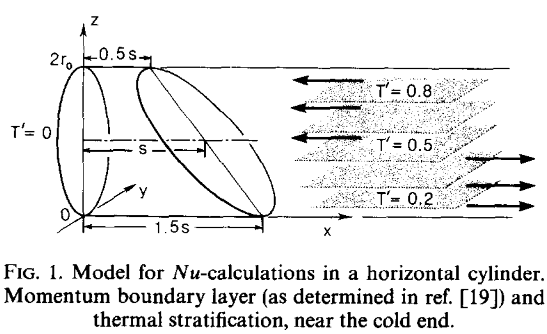

FIG. 1. Model for Nu-calculations in a horizontal cylinder. Momentum boundary layer (as determined in ref. [19]) and thermal stratification, near the cold end.

ESTIMATION OF THE NUSSELT NUMBER FOR THE CYLINDER

In order to evaluate Nu in the form

$$Nu = \frac{Q}{k\Delta TA/L}, \tag{4}$$

the total heat flux Q must be estimated. To calculate the heat flux to or from an end wall, the temperature gradient distribution at the wall must be known. Guided by our experiments [19], we apply equation (1) at the mid-height of the cylinder's cold end wall, and obtain for the interfacial temperature gradient at $z = r_0$

$$\left(\frac{dT'}{d(x/L)}\right)_{x/L=0} = \frac{m}{2}. \tag{5}$$

Since only the temperature gradient at $x/L = 0$ is important for the calculation of the local interfacial heat flux we replace equation (1) by the simpler relation

$$T' = \frac{m}{2}\frac{x}{L} \quad \text{for} \quad 0 \leqslant x/L \leqslant 1/m. \tag{6}$$

We define $d' \equiv 1/m$. Then, with $C = 0.80$, $h = 2r_0$, and $Ra_c = Ra$ one obtains from equation (2), after dimensionalizing, for the width of a 'thermal boundary layer' (TBL) at mid-height

$$d = 1.90 r_0\, Ra^{-0.25}. \tag{7}$$

Furthermore, we assume that the TBL width varies linearly with z, i.e. between $1.5d$ and $0.5d$ for $0 \leqslant z \leqslant 2r_0$, in analogy to the experimentally found momentum boundary layer behavior depicted in Fig. 1. For the temperature distribution outside the TBL we assume perfect linear stratification. For adiabatic walls at high Ra, this assumption is well supported by experiments and numerical modeling for shallow rectangular enclosures [6, 8, 12, 14] as well as by the aforementioned experiments on a long cylinder [17]. By summation over the local heat flux contributions, this simplifying model yields

$$Q = 0.61 k\Delta TA/d, \tag{8}$$

and

$$Nu = 0.32(L/r_0)Ra^{0.25}. \tag{9}$$

RESULTS AND DISCUSSION

Figure 2 shows a plot of equation (9) for $Ra \geqslant 3 \times 10^5$ (Curve 2), together with the experimental data of Kimura and Bejan [17]. The agreement between our simple model prediction and the experimental findings is rather good. Note that for a TBL of uniform thickness d the factor 0.61 in equation (8) changes to 0.50. Hence, only about 20% lower Q and Nu are obtained. This well illustrates the insensitivity of the Nu estimate to the exact TBL shape.

Probably the least realistic assumption in our Nu estimates is that of complete thermal stratification at the outer edge of the TBL. In reality, heat exchange between the fluid in the upper and lower half of the cylinder will lead to some inclination of the isotherms with respect to the horizontal mid-plane. The resulting deformation of the ideally stratified temperature profile at the TBL, however, appears to be small at high Ra's.

Bejan and Tien [2] have estimated the lower Ra-limit for the boundary layer regime of rectangular 2-D enclosures. Applied to the cylinder, this requires that $k_1 \leqslant 0.1 r_0/L$. Based on a more detailed analysis [22] this suggests that in our case the boundary layer regime extends to $Ra \approx 3 \times 10^5$. This has been taken, somewhat arbitrarily, as the lower limit for the representation of equation (9) in Fig. 2.

A possible dependence of Nu on Pr has not been considered in our model. As found for shallow rectangular enclosures [5, 13, 14] and for the cylinder [19], cells of recirculating fluid develop with increasing Ra in the vicinity of the end walls. Their appearance and physical character depend on aspect ratio, Ra and Pr, thus suggesting a dependence of Nu on Pr. At small Ra, Nu is generally taken as Pr-independent (as long as $Pr > 1$) as discussed in ref. [2].

Figure 2 also contains plots of the earlier theoretical predictions [15, 18]. Bejan and Tien's [15] analytical relation for Nu is

$$Nu = 1 + \frac{7}{46\,080}(Ra\, k_1)^2. \tag{10}$$

In the limit $Ra \to 0$, k_1 approaches r_0/L. We have plotted equation (10), as in the original paper, based on $k_1 = r_0/L$ (Curve 1).

As Shih's expression for Nu [18] is rather lengthy, it is not restated here. Also, his assumed velocity profiles for the end zones do not satisfy all boundary conditions, and do not obey the continuity equation. Furthermore, besides these underlying nonphysical assumptions, we feel that Shih's solution contains an error, since his Nu vs Ra curves cannot be reproduced from his relation. Thus we took Shih's curve for $r_0/L = 0.05$ (see his Fig. 5) and plotted it as Curve 3 in Fig. 2.

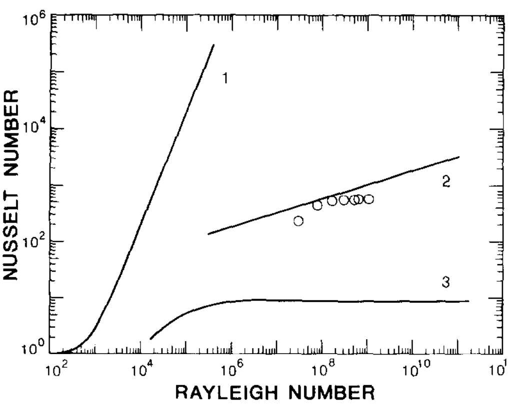

FIG. 2. Rayleigh number dependence of Nusselt number for a horizontal cylinder with an adiabatic wall and $r_0/L = 0.056$. Symbols: experimental data [17]. Curves: calculated, see text.

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